JEE–MAIN EXAMINATION – JANUARY 2026

(HELD ON FRIDAY 23rd JANUARY 2026) **TIME : 9:00 AM TO 12:00 NOON**

MATHEMATICS — TEST PAPER WITH ANSWER

SECTION-A

1. Let the domain of the function f(x) = $\log_3\log_5\log_7$ $(9x - x^2 - 13)$ be the interval (m, n). Let the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2} = 1$ have eccentricity $\frac{n}{3}$ and the length of the latus rectum $\frac{8m}{3}$. Then $b^2 - a^2$ is equal to :

(1) 5 (2) 11

(3) 9 (4) 7

Ans. (4)

Sol. $\log_5\left(\log_7\left(9x-x^2-13\right)\right)>0$

$\Rightarrow \quad 9x - x^2 - 13 > 7$

$x^2 - 9x + 20 < 0 \Rightarrow 4 < x < 5$

$m = 4, n = 5$

$\Rightarrow \quad e=\sqrt{1+\frac{b^2}{a^2}}=\frac{5}{3} \Rightarrow \frac{b^2}{a^2}=\frac{25}{9}-1=\frac{16}{9}$

$\frac{b}{a}=\frac{4}{3}$

$\Rightarrow \quad \frac{2b^2}{a}=\frac{8m}{3} \Rightarrow \frac{2b^2}{a}=\frac{32}{3}$

$\Rightarrow \quad 2b^2=\frac{32}{3}\times\frac{3b}{4} \Rightarrow b=4,\ a=3$

$b^2 - a^2 = 16 - 9 = 7$

2. Let f(x) = $\int\frac{\left(2-x^2\right).e^x}{\left(\sqrt{1+x}\right)(1-x)^{3/2}}dx$. If f(0) – 0, then $f\left(\frac{1}{2}\right)$ is equal to :

(1) $\sqrt{3e}-1$ (2) $\sqrt{2e}+1$

(3) $\sqrt{2e}-1$ (4) $\sqrt{3e}+1$

Ans. (1)

Sol. $\int e^x\left(\frac{(1-x^2)+1}{\sqrt{1+x}.(1-x)^{3/2}}\right)dx$

$\int e^x\left(\frac{(1-x^2)}{\sqrt{1+x}.(1-x)^{3/2}}+\frac{1}{\sqrt{1+x}.(1-x)^{3/2}}\right)dx$

$\int e^x\left(\sqrt{\frac{1+x}{1-x}}+\frac{1}{\sqrt{1+x}.(1-x)^{3/2}}\right)dx$

$=e^x\sqrt{\frac{1+x}{1-x}}+C$

$f(x)=e^x\sqrt{\frac{1+x}{1-x}}-1$

$f\left(\frac{1}{2}\right)=\sqrt{3e}-1$

3. Let $\vec{a}=-\hat{i}+\hat{j}+2\hat{k}$, $\vec{b}=\hat{i}-\hat{j}-3\hat{k}$, $\vec{c}=\vec{a}\times\vec{b}$ and $\vec{d}=\vec{c}\times\vec{a}$. Then $\left(\vec{a}-\vec{b}\right)\cdot\vec{d}$ is equal to :

(1) 4 (2) –4

(3) –2 (4) 2

Ans. (3)

Sol. $\vec{d}=\left(\vec{a}\times\vec{b}\right)\times\vec{a}$

$\vec{d}=(a^2)\vec{b}-(\vec{a}.\vec{b})\vec{a}$

$\vec{d}=6\vec{b}+8\vec{a}$

$\left(\vec{a}-\vec{b}\right).\vec{d}-=\left(\vec{a}-\vec{b}\right).\left(6\vec{b}+8\vec{a}\right)$

$= 8a^2-6b^2-2\vec{a}.\vec{b}$

$= 48 – 66 + 16 = –2$

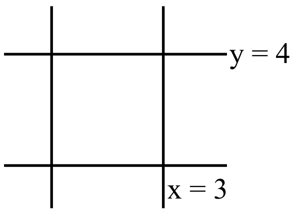
4. A rectangle is formed by the lines x = 0, y = 0, x = 3 and y = 4. Let the line L be perpendicular to 3x + y + 6 = 0 and divide the area of the rectangle into two equal parts. Then the distance of the point $\left(\frac{1}{2},-5\right)$ from the line L is equal to :

(1) $2\sqrt{5}$ (2) $3\sqrt{10}$

(3) $\sqrt{10}$ (4) $2\sqrt{10}$

Ans. **(4)**

Sol.



Line is $y = \frac{x}{3} + C$

Line passes thru $\left(\frac{3}{2},2\right)$

$2 = \frac{1}{2} + C \Rightarrow C = \frac{3}{2}$

$y = \frac{x}{3} + \frac{3}{2}$

$\Rightarrow 6y = 2x + 9$

Line is 2x – 6y + 9 = 0 &

Dist $= \left|\frac{1+30+9}{\sqrt{40}}\right| = \sqrt{40} = 2\sqrt{10}$

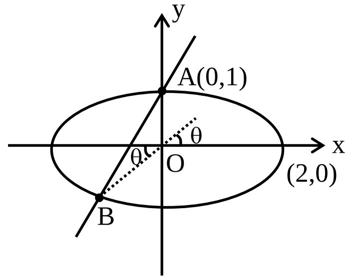
5. Let the line y – x = 1 intersect the ellipse $\frac{x^2}{2}+\frac{y^2}{1}=1$ at the points A and B. Then the angle made by the line segment AB at the center of the ellipse is :

(1) $\pi - \tan^{-1}\left(\frac{1}{4}\right)$ (2) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$

(3) $\frac{\pi}{2} + 2\tan^{-1}\left(\frac{1}{4}\right)$ (4) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{4}\right)$

Ans. **(2)**

Sol.



By solving line & equation of ellipse we get x = 0 & $x = -\frac{4}{3}$

$\therefore B\left(-\frac{4}{3},-\frac{1}{3}\right)$

$m_{OB} = \tan\theta = \frac{1}{4}$

$\because \angle AOB = \frac{\pi}{2} + \theta = \frac{\pi}{2} + \tan^{-1}\frac{1}{4}$

6. Let A={–2, –1, 0,1, 2, 3, 4}. Let R be a relation on A defined by xRy if and only if 2x+ y ≤ 2. Let *l* be the number of elements in R. Let m and n be the minimum number of elements required to be added in R to make it reflexive and symmetric relations respectively. Then *l* + m + n is equal to :

(1) 32 (2) 34

(3) 33 (4) 35

Ans. **(3)**

Sol. R{(–2, a), (–1, b), (0, c), (1, d), (2, e)}

a = {–2, –1, 0, 1, 2, 3, 4}; b = {–2, –1, 0, 1, 2, 3, 4}

c = {–2, –1, 0, 1, 2}; d = {–2, –1, 0}

e = {–2}

∴ No. of elements in R

= 7 + 7 + 5 + 3 + 1 = 23 = ℓ

Minimum number of element to be added to make it reflexive = m = 4 ⇒ {(1, 1), (2, 2), (3, 3), (4, 4)}

minimum number of element to be added to make it symmetric = n= 6

for 'n'

⇒ R = {(3, –2), (4, –2), (2, –1), (2, 0), (3, –1), (4, –1)}

∴ ℓ + m + n = 23 + 4 + 6 = 33

7. Let y = y(x) be the solution of the differential equation

$x^4dy + (4x^3y + 2\sin x)dx = 0,\ x > 0,\ y\left(\frac{\pi}{2}\right) = 0.$

Then $\pi^4 y\left(\frac{\pi}{3}\right)$ is equal to :

(1) 81 (2) 92

(3) 64 (4) 72

Ans. **(1)**

Sol. $(x^4dy + 4x^3ydx) = -2\sin x\,dx$

$\Rightarrow \int d(x^4y) = \int -2\sin x\,dx$

$\Rightarrow x^4y = 2\cos x + c$

$\Rightarrow x^4f(x) = 2\cos x + c$

As $f\left(\frac{\pi}{2}\right) = 0$

So, c = 0

$\left(\frac{\pi}{3}\right)^4 f\left(\frac{\pi}{3}\right) = 2\cos\frac{\pi}{3}$

$\pi^4 f\left(\frac{\pi}{3}\right) = 81$

8. If α and β (α < β) are the roots of the equation

$\left(-2+\sqrt{3}\right)\left(\left|\sqrt{x}-3\right|\right) + \left(x-6\sqrt{x}\right) + \left(9-2\sqrt{3}\right) = 0,$

$x \geq 0$, then $\sqrt{\frac{\beta}{\alpha}} + \sqrt{\alpha\beta}$ is equal to :

(1) 8 (2) 9

(3) 10 (4) 11

Ans. **(3)**

Sol. $\left(x-6\sqrt{x}+9\right)-\left(2-\sqrt{3}\right)\left|\sqrt{x}-3\right|-2\sqrt{3}=0$

$\Rightarrow \left|\sqrt{x}-3\right|^2-\left(2-\sqrt{3}\right)\left|\sqrt{x}-3\right|-2\sqrt{3}=0$

$\Rightarrow \left|\sqrt{x}-3\right|=2$ or $\left|\sqrt{x}-3\right|=-\sqrt{3}$ (not possible)

$\Rightarrow \sqrt{x} = 1$ or 5

$\Rightarrow x = 1$ or 5

$\Rightarrow \alpha = 1$ and $\beta = 25$

Aliter:

Let $x \geq 9$, let $\sqrt{x} = t \Rightarrow t \geq 3$

$(\sqrt{3}-2)(t-3)+(t-3)^2-2\sqrt{3}=0$

Let $t - 3 = u$

$u^2+(\sqrt{3}-2)u-2\sqrt{3}=0$

$u = 2$, or $u=-\sqrt{3}$

$\Rightarrow t-3=2$ or $t-3=-\sqrt{3}$

$\Rightarrow t = 5$ or $t=3-\sqrt{3}$ (rejected)

$\Rightarrow x = 25$

Now let $0 < x < 9$,

$-(\sqrt{3}-2)(t-3)+(t-3)^2-2\sqrt{3}=0$

let $t - 3 = u$

$u^2-(\sqrt{3}-2)u-2\sqrt{3}=0$

$\Rightarrow u=\sqrt{3}$ or $u=-2$

$\Rightarrow t=3+\sqrt{3}$ (rejected) or $t-3=-2$

$\Rightarrow t = 1 \Rightarrow x = 1$

$\alpha = 1, \beta = 25$

Now $\sqrt{\frac{\beta}{\alpha}}+\sqrt{\alpha\beta}=\sqrt{25}+\sqrt{25}=10$

9. Let $f(x) = \begin{cases} \dfrac{ax^2+2ax+3}{4x^2+4x-3}, & x \neq -\dfrac{3}{2}, \dfrac{1}{2} \\ b, & x = -\dfrac{3}{2}, \dfrac{1}{2} \end{cases}$

be continuous at $x = -\dfrac{3}{2}$. If fof(x) = $\dfrac{7}{5}$, then x is equal to :

(1) 2 (2) 1

(3) 0 (4) 1.4

Ans. (2)

Sol. $f(x) = \begin{cases} \dfrac{ax^2+2ax+3}{(2x-1)(2x+3)} & ; \ x \neq \dfrac{-3}{2}, \dfrac{1}{2} \\ b & ; \ x = \dfrac{-3}{2}, \dfrac{1}{2} \end{cases}$

for continuous at $x = \dfrac{-3}{2}$

LHL = RHL

$\Rightarrow \lim\limits_{x \to \frac{-3}{2}} \dfrac{(ax^2+2ax+3)}{(2x-1)(2x+3)}$

at $x = \dfrac{-3}{2} \Rightarrow$ Numerator = 0

$a\left(\dfrac{-3}{2}\right)^2 + 2a\left(\dfrac{-3}{2}\right) + 3 = 0$

$\dfrac{9}{4}a - 3a + 3 = 0$

$\dfrac{3a}{4} = 3 \Rightarrow a = 4$

$\therefore f(x) = \begin{cases} \dfrac{4x^2+8x+3}{(2x-1)(2x+3)} & ; \ x \neq \dfrac{-3}{2}, \dfrac{1}{2} \\ b & ; \ x = \dfrac{-3}{2}, \dfrac{1}{2} \end{cases}$

$f(x) = \begin{cases} \dfrac{(2x+1)(2x+3)}{(2x-1)(2x+3)} & ; \ x \neq \dfrac{-3}{2}, \dfrac{1}{2} \\ b & ; \ x = \dfrac{-3}{2}, \dfrac{1}{2} \end{cases}$

$fof(x) = f\left(\dfrac{2x+1}{2x-1}\right) = \dfrac{2\left(\dfrac{2x+1}{2x-1}\right)+1}{2\left(\dfrac{2x+1}{2x-1}\right)-1}$

$= \dfrac{6x+1}{2x+3} = \dfrac{7}{5}$

So x = 1 Ans.

10. The value of the integral $\displaystyle\int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \dfrac{dx}{1+\sqrt[3]{\tan 2x}}$ is :

(1) $\dfrac{\pi}{12}$ (2) $\dfrac{\pi}{18}$

(3) $\dfrac{\pi}{6}$ (4) $\dfrac{\pi}{3}$

Ans. (1)

Sol. $I = \displaystyle\int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \dfrac{dx}{1+\sqrt[3]{\tan 2x}}$ ...(1)

Apply king

$I = \displaystyle\int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \dfrac{dx}{1+\sqrt[3]{\tan 2\left(\dfrac{\pi}{4}-x\right)}}$

$= \displaystyle\int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \dfrac{dx}{1+\sqrt[3]{\cot 2x}}$ ...(2)

Add (1) + (2) $\Rightarrow 2I = \displaystyle\int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} (1)\,dx$

$I = \dfrac{1}{2}\left(\dfrac{\pi}{6}\right) = \dfrac{\pi}{12}$

11. Among the statements :

I : If $\begin{vmatrix} 1 & \cos\alpha & \cos\beta \\ \cos\alpha & 1 & \cos\gamma \\ \cos\beta & \cos\gamma & 1 \end{vmatrix} = \begin{vmatrix} 0 & \cos\alpha & \cos\beta \\ \cos\alpha & 0 & \cos\gamma \\ \cos\beta & \cos\gamma & 0 \end{vmatrix}$

, then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \dfrac{3}{2}$, and

II : If $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = px + q$, then

$p^2 = 196q^2$,

(1) both are false (2) only II is true

(3) both are true (4) only I is true

Ans. (1)

Sol. Let $\cos\alpha = x$

$\cos\beta = y$

$\cos\gamma = z$

$$\begin{vmatrix} 0 & x & y \\ x & 0 & z \\ y & z & 0 \end{vmatrix} = \begin{vmatrix} 1 & x & y \\ x & 1 & z \\ y & z & 1 \end{vmatrix}$$

Expending both sides, we get

$x^2 + y^2 + z^2 = 1$

i.e. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Statement 1 is false

Now,

$$\begin{vmatrix} x^2+x & 1+x & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = px+q$$

Put x = 0 both sides

$$q = \begin{vmatrix} 0 & 1 & -2 \\ -1 & 0 & -3 \\ 3 & -1 & -1 \end{vmatrix}$$

$\Rightarrow q = -12$

Now put x = 1 both sides

$$p+q = \begin{vmatrix} 2 & 2 & -1 \\ 4 & 3 & 3 \\ 6 & 1 & 1 \end{vmatrix} = 42$$

$\Rightarrow p = 54$

Now $\frac{p^2}{q^2} = \left(\frac{54}{-12}\right)^2 + 196$

$\Rightarrow p^2 \neq 196q^2$

Statement (2) is false

Correct option (1)

12. The value of $\frac{^{100}C_{50}}{51} + \frac{^{100}C_{51}}{52} + \ldots + \frac{^{100}C_{100}}{101}$ is :

(1) $\frac{2^{101}}{100}$ (2) $\frac{2^{100}}{100}$

(3) $\frac{2^{101}}{101}$ (4) $\frac{2^{100}}{101}$

Ans. **(4)**

Sol. $S = \sum_{r=50}^{100} \frac{^{100}C_r}{r+1} = \sum_{r=50}^{100} \frac{1}{r+1} \cdot \frac{r+1}{101} \cdot {}^{101}C_{r+1}$

$S = \frac{1}{101} \sum_{r=50}^{100} {}^{101}C_{r+1}$

$= \frac{1}{101} \times \frac{2^{101}}{2} = \frac{2^{100}}{101}$

Option (4)

13. Let the mean and variance of 8 numbers –10, –7, –1, x, y, 9, 2, 16 be $\frac{7}{2}$ and $\frac{293}{4}$, respectively. Then the mean of 4 numbers x, y, x + y + 1, |x – y| is:

(1) 11 (2) 9

(3) 10 (4) 12

Ans. **(1)**

Sol. Mean $= \frac{-18+x+y+2+9+16}{8} = \frac{7}{3}$

$= \frac{x+y+9}{8} = \frac{7}{2} \Rightarrow x+y+9 = 28$ …..(1)

Variance $= \frac{\sum z_i^2}{8} - (\mu)^2 = \frac{293}{4}$

$\Rightarrow \frac{10^2+7^2+1^2+x^2+y^2+2^2+9^2+16^2}{8} - \left(\frac{7}{2}\right)^2 = \frac{293}{4}$ …. (2)

Solving (1) & (2) $\Rightarrow$ x = 12, y = 7

Mean of (1 +x + y), x, y, |y –x| is

$\Rightarrow \frac{20+12+7+5}{4} = \frac{44}{4} = 11$

Option (1)

14. The sum of all possible values of $n \in N$, so that the coefficients of x, x^2 and x^3 in the expansion of $(1 + x^2)^2 (1 + x)^n$, are in arithmetic progression is :

(1) 3 (2) 7

(3) 12 (4) 9

Ans. (4)

Sol. $(x^4 + 2x^2 + 1)\,({}^nC_0x^0 + {}^nC_1x^1 + {}^nC_2x^2 + {}^nC_3x^3 + \ldots)$

Coefficient $x \Rightarrow {}^nC_1$,

coeff. of $x^2 \Rightarrow 2 + {}^nC_2$

$$2 + \frac{n(n-1)}{2}$$

Coeff. of $x^3 = 2.{}^nC_1 + {}^nC_3$

$$= 2n + \frac{n(n-1)(n-2)}{6} \quad (\text{if } x \ge 3)$$

Now according to question

$$n + 2n + \frac{n(n-1)(n-2)}{6} = 2\left[2 + \frac{n(n-1)}{2}\right]$$

$$3n + \frac{n(n-1)(n-2)}{6} = 4 + n(n-1)$$

$\Rightarrow n^3 - 9n^2 + 26n - 24 = 0$

$\Rightarrow n = 2, 3, 4 \qquad \Rightarrow n = 3, 4$

Now checking for $n = 2$

$$\left.\begin{array}{l}\text{Coeff. of } x = 2\\ \text{Coeff. of } x^2 = 3\\ \text{Coeff. of } x^3 = 4\end{array}\right\} \Rightarrow \text{are in A.P.}$$

$\Rightarrow n = 2$ is also the correct choice

Required sum of values of 'n'

$= 2 + 3 + 4 = 9$

Option (4)

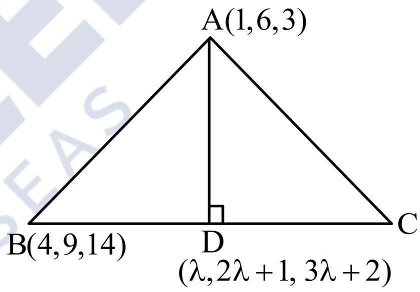
15. The vertices B and C of a triangle ABC lie on the line $\frac{x}{1} = \frac{1-y}{-2} = \frac{z-2}{3}$. The coordinates of A and B are (1, 6, 3) and (4, 9, α) respectively and C is at a distance of 10 units from B. The area (in sq. units) of ΔABC is:

(1) $5\sqrt{13}$ (2) $15\sqrt{13}$

(3) $20\sqrt{13}$ (4) $10\sqrt{13}$

Ans. (1)

Sol. $\frac{4}{1} = \frac{9-1}{2} = \frac{\alpha - 2}{3} \Rightarrow \alpha = 14$



$$\overrightarrow{AD}.(\hat{i} + 2\hat{j} + 3\hat{k}) = 0$$

$$(\lambda - 1)\hat{i} + (2\lambda - 5)\hat{j} + (3\lambda - 1)\hat{k} = \overrightarrow{AD}$$

$\Rightarrow \lambda - 1 + 4\lambda - 10 + 9\lambda - 3 = 0$

$\Rightarrow 14\lambda = 14 \Rightarrow \lambda = 1$

$D = (1, 3, 5)$

$AD = \sqrt{3^2 + 2^2} = \sqrt{13}$

$\text{Ar}(\Delta ABC) = \frac{1}{2} \times \sqrt{13} \times 10 = 5\sqrt{13}$

Option (A)

16. Number of solutions of

$\sqrt{3}\ \cos 2\theta + 8\cos\theta + 3\sqrt{3} = 0,\ \theta \in [-3\pi, 2\pi]$ is:

(1) 0 (2) 5

(3) 3 (4) 4

Ans. (2)

Sol. $\sqrt{3}\left(2\cos^2\theta - 1\right) + 8\cos\theta + 3\sqrt{3} = 0$

$2\sqrt{3}\cos^2\theta + 8\cos\theta + 2\sqrt{3} = 0$

$\left(\sqrt{3}\cos + 1\right)\left(\cos\theta + \sqrt{3}\right) = 0$

$\cos\theta = -\dfrac{1}{\sqrt{3}}$

as $-\sqrt{3}$ (reject)

$\therefore$ θ = will have 5 value in [–3π, 2π]

Ans. = 5 ⇒ option (2)

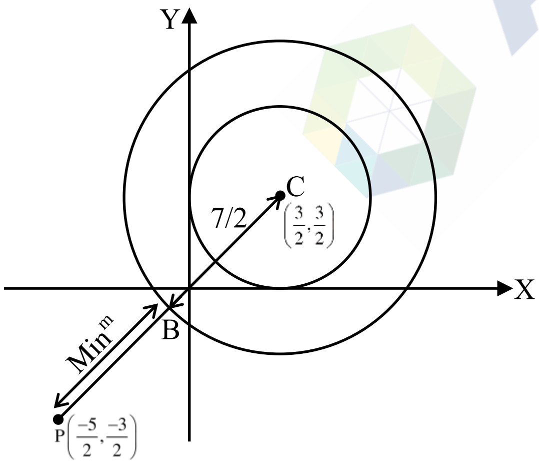
17. Let S = {z : 3 ≤ | 2z – 3(1 + i)| ≤ 7} be a set of complex numbers. Then $\underset{z\in S}{\text{Min}}\left|\left(z + \dfrac{1}{2}(5+3i)\right)\right|$ is equal to :

(1) $\dfrac{1}{2}$ (2) $\dfrac{3}{2}$

(3) 2 (4) $\dfrac{5}{2}$

Ans. (2)

Sol. $\dfrac{3}{2} \le \left|z - \dfrac{3}{2}(1+i)\right| \le \dfrac{7}{2}$



$\underset{z\in S}{\text{Min}}\left|z - \left(\dfrac{-5}{2} - \dfrac{3}{2}i\right)\right| = PB$

$PB = PC - \dfrac{7}{2} \Rightarrow 5 - \dfrac{7}{2} \Rightarrow \dfrac{3}{2}$

Option (2)

18. Let α and β respectively be the maximum and the minimum values of the function

$$f(\theta) = 4\left(\sin^4\left(\frac{7\pi}{2} - \theta\right) + \sin^4(11\pi + \theta)\right)$$

$$-2\left(\sin^6\left(\frac{3\pi}{2} - \theta\right) + \sin^6(9\pi - \theta)\right),\ \theta \in \mathbf{R}.$$

Then α + 2β is equal to :

(1) 4 (2) 5

(3) 3 (4) 6

Ans. (2)

Sol. $f(\theta) = 4\left(\sin^4\left(\frac{7\pi}{2} - \theta\right) + \sin^4(11\pi + \theta)\right) - 2\left(\sin^6\left(\frac{3\pi}{2} - \theta\right) + \sin^6(9\pi - \theta)\right)$

$f(\theta) = 4(\cos^4(\theta) + \sin^4(\theta)) - 2(\cos^6\theta + \sin^6\theta)$

$f(\theta) = 4(1 - 2\sin^2\theta\cos^2\theta) - 2(1 - 3\sin^2\theta\cos^2\theta)$

$f(\theta) = 2 - 2\sin^2\theta\cos^2\theta$

$f(\theta) = 2 - \dfrac{\sin^2(2\theta)}{2}$

$\alpha = f(\theta)_{max} = 2$

$\beta = f(\theta)_{min} = \dfrac{3}{2}$

$\Rightarrow \alpha + 2\beta = 5$

Ans. = 5 option (2)

19. A building construction work can be completed by two masons A and B together in 22.5 days. Mason A alone can complete the construction work in 24 days less than mason B alone. Then mason A alone will complete the construction work in:

(1) 24 days (2) 42 days

(3) 30 days (4) 36 days

Ans. (4)

Sol. Let time taken by mason a alone to complete the work be in x days so, mason B along take x + 24 days

work done by A in 1 day $= \frac{1}{x}$

work done by B in 1 day $= \frac{1}{x+24}$

so work done by A + B in 1 day $= \frac{1}{22.5}$

So, $\frac{1}{x}+\frac{1}{x+24}=\frac{2}{45}$

$x^2 - 21x - 540 = 0$

$x = 36$ &

$x = -15$ (rejected)

Ans. = 36 option (4)

20. Let the direction cosines of two lines satisfy the equations : $4\ell + m - n = 0$ and $2mn + 10n\ell + 3\ell m = 0$.

Then the cosine of the acute angle between these lines is :

(1) $\frac{10}{\sqrt{38}}$ (2) $\frac{20}{3\sqrt{38}}$

(3) $\frac{10}{7\sqrt{38}}$ (4) $\frac{10}{3\sqrt{38}}$

Ans. **(4)**

Sol. Direction cosines of two lines satisfy the equation

$\Rightarrow 4\ell + m - n = 0$ ...(1)

$2mn + 10n\ell + 3\ell m = 0$ ...(2)

& we know

$\Rightarrow \ell^2 + m^2 - n^2 = 1$ ...(3)

$\Rightarrow n = 4\ell + m$ putting in eqn. (1)

$\Rightarrow n(2m + 10\ell) + 3\ell m = 0$

$\Rightarrow (4\ell + m)(2m + 10\ell) + 3\ell m = 0$

$\Rightarrow 8\ell m + 40\ell^2 + 2m^2 + 10\ell m + 3\ell m = 0$

$\Rightarrow 40\ell^2 + 21\ell m + 2m^2 = 0$

$\Rightarrow (8\ell + m)(5\ell + 2m) = 0$

Case 1 : $8\ell + m = 0 \Rightarrow m = -8\ell$

Case 2 : $5\ell + 2m = 0 \Rightarrow m = \frac{-5}{2}\ell$

So direction ratio of L_1 is $\ell, -8\ell, -4\ell$

& direction ratio of L_2 is $\ell, \frac{-5\ell}{2}, \frac{3\ell}{2}$

$$\cos\theta = \left|\frac{\ell^2 + 20\ell^2 - 6\ell^2}{\sqrt{\ell^2 + 64\ell^2 + 16\ell^2}\sqrt{\ell^2 + \frac{25\ell^2}{4} + \frac{9\ell^2}{4}}}\right|$$

$$= \frac{15\ell^2}{(9\ell)\frac{\sqrt{38}\,\ell}{2}} = \frac{10}{3\sqrt{38}}$$

Ans. $= \frac{10}{3\sqrt{38}}$

SECTION-B

21. Let $|A| = 6$, where A is a 3×3 matrix. If $|adj(3adj(A^2 \cdot adj(2A)))| = 2^m.3^n$, m, n $\in$ **N**, then m + n is equal to ________.

Ans. (62)

Sol. $adj\ 2A = 2^2\ adj\ A \quad \because adj\ kA = k^{n-1}\ (adj\ A)$

$= 4\ adj\ A$

Now $A^2\ (adj\ 2A) = 4A\ (adj\ A)$

$= 4\ A\ |A|\ I_3$

$= 24A$

Now $3\ adj\ (A^2(adj\ 2A)) = 3\ adj\ (24\ A)$

$= 3.(24)^2\ adj\ A$

Now $\left|adj\left(3adj\ (A^2(adj 2A))\right)\right|$

$= \left|adj\left(3.(24)^2 adj A\right)\right|$

$= \left|\left(3.(24)^2\right)^2 (adj\ adj A)\right|$

$= \left|3^6.2^{12} adj\ adj A\right|$

$= (3^6.2^{12})^3\ |adj\ adj\ A|$

$= 3^{18}.2^{36}.(A)^4$

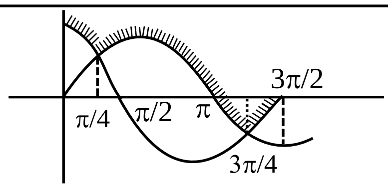
$3^{22}.2^{40}$

$\therefore\ m + n = 62$

22. Let the area of the region bounded by the curve $y = \max\{\sin x, \cos x\}$, lines $x = 0$, $x = \frac{3\pi}{2}$, and the x-axis be A. Then, $A + A^2$ is equal to

Ans. (12)

Sol.



$$A = \int_0^{\pi/4} \cos x\,dx + \int_{\pi/4}^{\pi} \sin x\,dx + \int_{\pi}^{5\pi/4} -\sin x\,dx + \int_{5\pi/4}^{3\pi/2} -\cos x\,dx$$

$$A = (\sin x)_0^{\pi/4} + (\cos x)_{\pi}^{\pi/4} + (\cos x)_{\pi}^{5\pi/4} + (\sin x)_{3\pi/2}^{5\pi/4}$$

$$A = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + 1 + 1 - \frac{1}{\sqrt{2}} + 1 - \frac{1}{\sqrt{2}} = 3$$

$A^2 + A = 12$

23. Let f be a twice differentiable non-negative function such that

$(f(x))^2 = 25 + \int_0^x \left((f(t))^2 + (f'(t))^2\right)dt$. Then the mean of $f(\log_e(1))$, $f(\log_e(2))$, ..., $f(\log_e(625))$ is equal to ________ .

Ans. (1565)

Sol. $2f(x)f'(x) = f^2(x) + (f'(x))^2$

$\Rightarrow (f(x) - f'(x))^2 = 0$

$\Rightarrow f(x) = f'(x)$

$\Rightarrow \ell n(f(x)) = x + c \Rightarrow f(x) = c'e^x$

$f(0) = 5 \Rightarrow f(x) = 5e^x$

$$\text{Mean} = \frac{f(\ell n1) + f(\ell n2) + \ldots\ldots\ldots + f(\ell n625)}{625}$$

$$= \frac{5[1 + 2 + \ldots\ldots\ldots\ldots + 625]}{625} = 1565$$

24. From the first 100 natural numbers, two numbers first a and then b are selected randomly without replacement. If the probability that $a - b \geq 10$ is $\frac{m}{n}$, gcd (m, n) = 1, then m + n is equal to _______.

Ans. **(311)**

Sol. $a - b \geq 10$

Total cases = 100×99

Fav. Cases = $1 + 2 + 3 + \ldots 90$

$$\text{Req. Prob} = \frac{1+2+\ldots+90}{100\times 99}$$

$$\frac{m}{n} = \frac{90\left(\frac{91}{2}\right)}{100(99)} = \frac{91}{220}$$

$m + n = 311$

25. The number of 4-letter words, with or without meaning, which can be formed using the letters PQRPQRSTUVP, is _______ .

Ans. **(1422)**

Sol. $P \to 3, Q \to 2, R \to 2,\ \ S,T,U,V$

Case I 3 alike, 1 different

$$^{1}C_{1} \times {}^{6}C_{1} \times \frac{4!}{3!} = 24$$

Case II 2 alike, 2 alike

$$^{3}C_{2} \times \frac{4!}{2!2!} = 18$$

Case III 2 alike, 2 different

$$^{3}C_{1} \times {}^{6}C_{2} \times \frac{4!}{2!} = 540$$

Case IV All 4 different

$$^{7}C_{4} \times 4! = 840$$

Total words = 1422